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X. Litrico, V. Guinot

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Discussion of “Development and Verification of an Analytical Solution for Forecasting Nonlinear Kinematic Flood Waves” by Sergio E. Serrano

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Xavier Litrico¹ and Vincent Guinot²

This discussion appeared in the Journal of Hydrologic Engineering, Vol. 12, No. 6, pp. 703-705.

The author presents an interesting method to forecast nonlinear kinematic flood waves (Serrano, 2006). As a first remark, the discussers would like to stress the fact that the proposed method is an approximate solution and not an analytical solution, as suggested by the title of the paper. Indeed, an analytical expression is given by the backwards characteristics methods, as in Eq. (6) of the paper. However, since this solution is implicit, the author proposes an approximate explicit solution by making simplifying assumptions.

The discussers would like to raise two points in this discussion:

- 1) There exists a forward, analytical solution to the Kinematic Wave Equation (KWE). The range of time lags for which this analytical solution is applicable being restricted, a linearization of the KWE is a good compromise between accuracy and robustness. However,
- 2) The KWE should be linearized in a such a way that the propagation speed in the original and linearized equations remain comparable. In this respect, the comparison between the nonlinear KWE and the linearized KWE as provided in Serrano (2006) is unfair because it is based on an improper linearization.

These points are treated separately in what follows. We only focus on the case with zero lateral discharge $q = 0$. The results of this discussion also apply with slight modifications when the lateral discharge is not zero.

A Forward Analytical Solution to the Nonlinear Kinematic Wave

The characteristic equation presented in the paper is to be solved

$$\frac{dQ}{dt} = 0 \text{ for } \frac{dt}{dx} = \alpha\beta Q^{\beta-1} \quad (1)$$

Therefore a forward solution method gives

$$Q(x, t + T(x, t)) = Q(0, t) \quad (2)$$

where the distance x and the time lag $T(x, t)$ are related by

$$T(x, t) = x \frac{dt}{dx} = x\alpha\beta Q(0, t)^{\beta-1} \quad (3)$$

¹Researcher, UMR Gestion de l'Eau, Acteurs, Usages, Cemagref, 361, rue JF Breton, B.P. 5095, 34196 Montpellier Cedex 5, France. E-mail: xavier.litrico@cemagref.fr

²Lecturer, Hydrosiences Montpellier UMR 5569 (CNRS, IRD, UM1, UM2), Maison des Sciences de l'Eau, 34095 Montpellier Cedex 5, France. E-mail: guinot@msem.univ-montp2.fr

This solution given by Eqs. (2–3) of this discussion is exact, explicit and does not require any approximate solution procedure. The downstream discharge is not computed at regular time instants, but a simple linear interpolation can be done to obtain the discharge at the required time instants.

This solution is limited to time delay variations that do not yield shocks, i.e. such that $\partial Q/\partial t$ remains finite.

The simulation results of this discussion are obtained with this simple and direct procedure.

A Fair Comparison between Linear and Nonlinear Models

In the paper, the author compares the flood hydrographs simulated with various values of a parameter β , which is given from the stage discharge relationship of the river $A = \alpha Q^\beta$. The author observes that “For values of $\beta < 1$, the peak flow occurs at a time earlier than that predicted by the linear hydrograph ($\beta = 1$). For values of $\beta > 1$, the peak flow occurs at a time later than that predicted by the linear hydrograph.” And he concludes by “This suggests that, depending on the stream parameters, the simulation of flood propagation may not be appropriately described by a linear model.”

The discussers do not agree with the author concerning this last sentence. From our viewpoint, one issue is the nonlinear dynamic behavior of the flow propagation in the river; another one is the ability of a linear model to represent flow propagation. Indeed, it is well-known that the large majority of physical system has a nonlinear behavior. However, linear models are widely used to represent the dynamics of a nonlinear system *around a functioning point*.

In the case considered here, a fair comparison between linear and nonlinear models would be to *linearize* the nonlinear model around a functioning point (a reference discharge) to obtain a linear model.

The linearization of the kinematic wave equation given by Eq. (1) of the paper around a reference discharge \bar{Q} is obtained by assuming $Q = \bar{Q} + \delta Q$ and neglecting second order terms, which leads to the linear kinematic wave equation:

$$\frac{\partial \delta Q}{\partial x} + \alpha \beta \bar{Q}^{\beta-1} \frac{\partial \delta Q}{\partial t} = 0 \quad (4)$$

Then, the flood routing represented by Eq. (4) of this discussion is equivalent to

$$\delta Q(L, t) = \delta Q(0, t - \bar{T}) \quad (5)$$

where \bar{T} is a constant time-delay, given by:

$$\bar{T} = \alpha \beta L \bar{Q}^{\beta-1} \quad (6)$$

and L is the length of the river.

From the original paper, the average time delay obtained using $\beta = 1$ is:

$$\bar{T} = \alpha L \quad (7)$$

This may be coherent with equation (6) of this discussion as long as the reference discharge \bar{Q} verifies:

$$\bar{Q} = \left(\frac{1}{\beta} \right)^{\frac{1}{\beta-1}} \quad (8)$$

However, in this relation, \bar{Q} is not related to the input discharge, and may lead to very low values.

To summarize, the relationship $A = \alpha Q^\beta$ presented in the original paper has two degrees of freedom, namely the coefficient α and the exponent β . In an appropriate linearization procedure, the simplification induced by the modification of β must be compensated by a modification of the coefficient α . More generally, any modification of β can be accompanied with an appropriate modification of α such that the propagation speed of the wave remains a constant about a reference discharge \bar{Q} . From Eq. (2), changing β to β' keeps the travel time \bar{T} unchanged provided that α is changed to a value α' that satisfies the following condition:

$$\alpha' \beta' \bar{Q}^{\beta'-1} = \alpha \beta \bar{Q}^{\beta-1} \quad (9)$$

In the particular case of a linearization, $\beta' = 1$ and Eq. (9) yields the following expression for α' :

$$\alpha' = \alpha \beta \bar{Q}^{\beta-1} \quad (10)$$

Such a modification of coefficient α would provide a correct linear simulation, contrarily to the ones presented in the original paper.

As an example, Figure 1 shows the advection of a hydrograph along a river reach of 10 km for $\alpha = 3 \text{ m}^{2-3\beta} \text{ s}^\beta$ and $\beta = 1.5$. As indicated by Eq. (10), the nonlinear KWE can be linearized using the reference discharge $\bar{Q} = 1.5 \text{ m}^3/\text{s}$. In this case Eq. (10) gives a value $\alpha' = 5.51 \text{ m}^{-1} \text{ s}$. The solution given by the linearization is indicated by the plus-shaped markers in Figure 1. It is easy to see that the propagation speed of the reference discharge $\bar{Q} = 1.5 \text{ m}^3/\text{s}$ is preserved by the linearization. On the contrary, the hydrograph obtained by using $\beta = 1$ leads to underestimate the time delay.

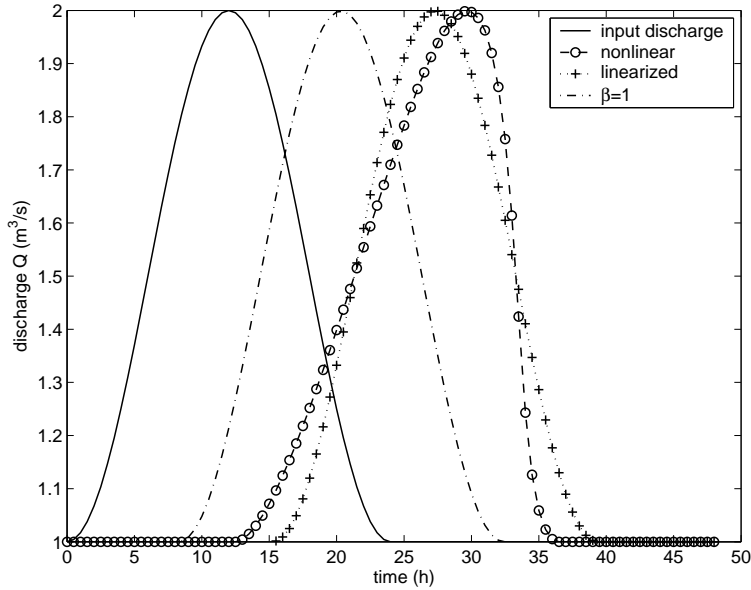


Figure 1: Comparison of different models for the propagation of a sinusoidal hydrograph on a 10 km long river with $\alpha = 3.0 \text{ m}^{2-3\beta} \text{ s}^\beta$, $\beta = 1.5$ and $\bar{Q} = 1.5 \text{ m}^3/\text{s}$.

The value of β mainly has an effect on the symmetry of the hydrograph. Values of β larger than unity tend to smooth out the front and steepen the tail (Figure 1), while values of β

smaller than unity lead to steepen the head and smooth out the tail of the hydrograph (see Figure 2, obtained for $\alpha = 3.0 \text{ m}^{2-3\beta}\text{s}^\beta$ and $\beta = 0.3$). From a physical point of view, $\beta < 1$ is a more realistic representation of most natural and artificial channels.

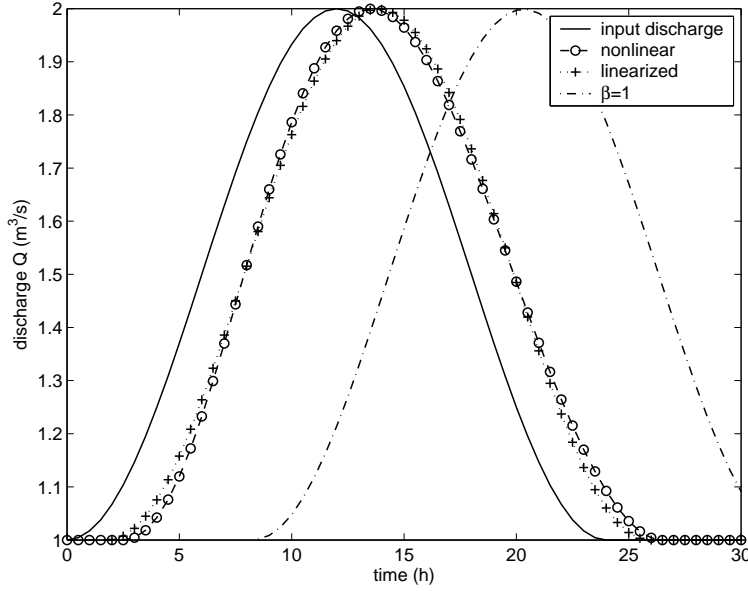


Figure 2: Comparison of different models for the propagation of a sinusoidal hydrograph on a 10 km long river with $\alpha = 3.0 \text{ m}^{2-3\beta}\text{s}^\beta$, $\beta = 0.3$ and $\bar{Q} = 1.5 \text{ m}^3/\text{s}$.

Choice of the Reference Discharge \bar{Q}

Then, in order to perform a fair comparison between linear and nonlinear models, we need to determine the suitable reference discharge \bar{Q} . This may be done by two means:

- A priori evaluation of \bar{Q} , by choosing the average input discharge, or the maximum input discharge, or another one compatible with the input data,
- A posteriori evaluation of \bar{Q} , by estimating the average time delay of the river based on available measurements, and computing the corresponding average discharge by inverting Eq. (6) of this discussion.

The second method can be combined with an optimization procedure. In this case, one looks for the value of the constant time delay (and the corresponding reference discharge) that minimizes a chosen criteria. Fig. 3 plots the fit coefficient γ proposed in Eq. (26) of the paper for linear simulations with various reference discharges \bar{Q} , for two river reaches of 10 km, represented by the nonlinear KWE, one with $\beta = 0.3$ and the other one with $\beta = 1.5$.

The circular dots represent the value of the fit coefficient γ obtained by choosing $\beta = 1$ as did the author in his paper. The reference discharge is then given by Eq. (8). It clearly appears that a model properly linearized leads to a better fit. In the case considered here, a reference discharge $\bar{Q} = 1.5 \text{ m}^3/\text{s}$ is a good choice in terms of this fit coefficient γ .

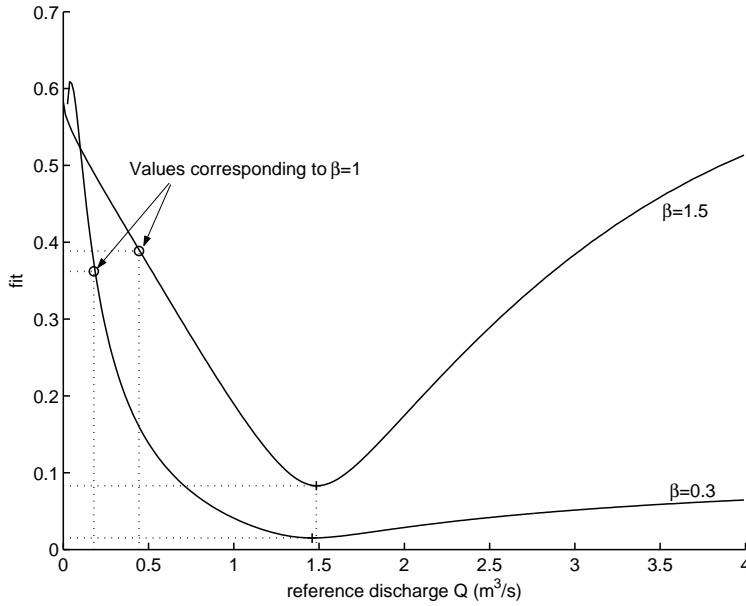


Figure 3: Fit coefficient γ for the propagation of a sinusoidal hydrograph on a river of length $L = 10$ km represented by the nonlinear KWE with $\alpha = 3 \text{ m}^{2-3\beta} \text{ s}^\beta$, $\beta = 0.3$ and $\beta = 1.5$.

Application to the Schuylkill River

The application section of the paper considers the Schuylkill river, for which the parameters are given by $\alpha = 4.6 \text{ m}^{2-3\beta} \text{ s}^\beta$, $\beta = 0.594$, and $L = 21$ km. The daily discharge measurements were retrieved from the USGS website (<http://waterdata.usgs.gov/wis/>) and were used to compute the time delay $T(L, t)$ given by Eq. (3) of this discussion corresponding to the input data for Fig. 6 of the original paper. This is plotted in Fig. 4. It appears that the delay $T(L, t)$ varies from 1 h to 4.5 h, and that an average value \bar{T} of about 2 h should lead to an adequate linear approximation of the dynamic behaviour of the river. Using Eq. (6), this corresponds to a reference discharge of $\bar{Q} = 166 \text{ m}^3/\text{s}$.

As a comparison, the time delay chosen by the author in his linear simulation corresponds to a delay of $\alpha L = 4.6 \times 21000 = 26.8$ h, which is more than ten times the average delay of the river! And the corresponding reference discharge is given by Eq. (8) $\bar{Q} = 0.277 \text{ m}^3/\text{s}$, which is about 100 times smaller than the discharges of figures 5 and 8 of the paper.

It is therefore not surprising that the simulations with a linear model corresponding to such a large delay lead to a poor fit coefficient.

Finally, it is recalled that the KWE is valid under the assumptions of negligible inertia and velocities, assuming small depths and steep slopes, so that the energy grade line can be assumed identical to the river bed slope. With an average slope smaller than 0.1 % between the two gauging stations, this assumption is obviously not valid in the case of the Shuylkill river between Norristown and Philadelphia.

As a conclusion, the backward method proposed by the author to simulate the nonlinear kinematic wave is not analytical, while the discussers provide a forward analytical solution. The discussers argue that a linear kinematic wave model may be adequate to simulate nonlinear kinematic river flow, provided that a proper linearization is done.

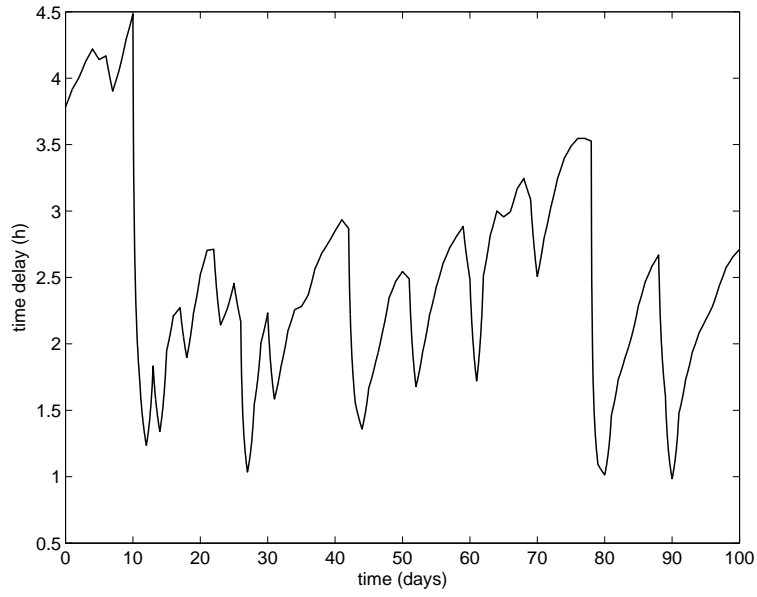


Figure 4: Variation of the time delay along time $T(L, t)$ for daily input discharge of the Schuylkill river at Norristown

References

Serrano, S. E. (2006). Development and verification of an analytical solution for forecasting nonlinear kinematic flood waves. *Journal of Hydrologic Engineering*, 11:347–353.